Mall Boom or Bust
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Our Presentation

- What is a mall
- Discrete logistic growth model
- Assumptions we made
- Our model
- Our findings
- Our conclusion
What is a Mall?

- A collection of independent retail stores, services, and a parking area conceived, constructed, and maintained by a management firm as a unit.
- Shopping malls may also contain restaurants, banks, theatres, professional offices, service stations, and other establishments.
Thunderbird

- Located in Menomonie, WI
London Square

- Located in Eau Claire, WI
- Younker’s
Oakwood

- Located in Eau Claire, WI
- 8 million visits per year
- 130 stores
- Key Attractions:
  - Department Stores
  - Women's Apparel
  - Housewares & Home
  - Books & Entertainment
  - Movie Theater
  - Food Court and Restaurant
Mall of America

- Located in Bloomington, MN
- Currently the largest fully enclosed retail and entertainment complex in the United States.
- More than 520 stores
- 600,000 to 900,000 weekly visits depending on season
- Nearly $1.5 billion annually income
Discrete Logistic Growth Model
Population Model

- \( X(n) = \) population of the mall at year \( n \)

- \( r = \) the intrinsic growth rate of the stores

- The difference between the current and previous year is represented by the equation:

\[
X(n + 1) - X(n) = rX(n)
\]
Population Model (cont.)

- The population for the next year would be represented by the equation:
  \[ X(n+1) = RX(n) \text{ where } R = r + 1 \]

- Our model assumes that the growth rate is dependant on the population. So, growth rate would be represented by \( r(x) \).
Carrying Capacity

The carrying capacity of the store population would be the maximum number of stores possible given current space restrictions. The carrying capacity is represented by a constant $K$. 
Ockham’s Razor

- If there are several possible explanations for some observation, and no significant evidence to judge the validity of those hypotheses, you should always use the simplest explanation possible.

- Also known as the principle of parsimony – scientists should make no more assumptions or assume no more causes than are absolutely necessary to explain their observations.
By Ockham’s Razor

- Growth rate would be linear (of the form $r(x) = mx + b$)
- $r(0) = \rho$ (an intrinsic growth rate without regard to restrictions like space)
By Ockham’s Razor (cont.)

- $r(K) = 0$ (no growth)

- $r(x) = -(\rho /K)x + \rho$
- $r(X(n)) = -(\rho /K)x + \rho$
Basic Logistic Population Model

- $X(n+1) - X(n) = \left[-\left(\frac{\rho}{K}\right)x + \rho\right]X(n)$
- $X(n+1) = \left[-\left(\frac{\rho}{K}\right)x + \rho\right]X(n) + X(n)$
- $X(n+1) = X(n)\left[1 + \rho(1-X(n)/K)\right]$
Steady State

- A steady state is a point where an system “likes” to remain once reached.
- The fundamental equation $X(n+1) = f(X(n))$ is a 1st order recurrence equation.
- To find the steady states of our model solve the following equation for $X$:

  $$X[1+ \rho(1-X(n)/K)] = X$$

  $$X = 0 \text{, } X = K$$
Steady State (cont.)

- Essentially, once the mall reaches capacity it has will most likely remain full.
- Conversely, once a mall becomes vacant it is highly unlikely that any stores will be attracted to the location.
Stability

- Stability is the tendency to approach a steady state.
- To determine stability, find the derivative of \( f(x) = X[1 + \rho(1-X(n)/K)] \)
  - Which is: \( f'(x) = 1 + \rho - (2 \rho /K)X \)
- Stable if \( |f'(x)| < 1 \)
Stability (cont.)

Findings:

\[
\begin{align*}
\text{f'}(0) & : f'(0) = 1 + \rho \\
& \quad |1 + \rho| < 1 \\
& \text{(0 is an unstable fixed point)}
\end{align*}
\]

\[
\begin{align*}
\text{f'}(K) & : f'(K) = 1 + \rho - 2 \rho \\
& = |1 - f| < 1 \\
& -1 < 1 - \rho < 1 \\
& 0 < \rho < 2
\end{align*}
\]

If the intrinsic growth rate is out of range, we find chaotic behavior in the model.
Assumptions
Assumptions We Made

- The mall is a fixed size and location
- In our model we will be considering customers, stores, and mall management.
Assumptions (cont.)

- Mall management rationally and intentionally controls what they charge for rent in an effort to get a maximum profit for the mall.
- Stores pass rent off to the customer within the prices of the products they sell.
Assumptions (cont.)

■ Symbiosis
  ■ Population of customers and stores are positively associated.
  ■ If one increases or decreases the other follows until they reach capacity.

■ Finite Carrying Capacity
  ■ There is a maximum number of customers and stores a mall can have.
Laws of economics

- Supply is positively associated with the price.
- Demand is negatively associated with the price.
Opportunistic Rent

- Year n-1
  - stores make a profit

- Year n
  - mall management increases the rent to maximize their profit
  - stores pass off the increase of rent to the customers by increasing prices
Opportunistic Rent (cont.)

- Year n+1
  - A noticeable loss in customers will be observed and store will lose profit
- Year n+2
  - stores will leave if not making a profit
  - mall management will have to decrease the rent to keep stores or get new stores to move in
- This cycle will continue until mall management and the stores both reach an agreeable opportunistic rent.
Misc. Factors Not Considered

- Niche effectiveness (different types of stores)
- Price elasticity (insensitivity to price change)
- Economies of scale (more variety)
- Population of surrounding area
- Attractiveness of the mall
Our Model
Formulating the Mall Model

- Let $X(n)$ be the population of mall customers at year $n$.
- Let $Y(n)$ be the number of stores in the mall at year $n$.
- Let $K$ be the mall carrying capacity of stores.
The Customers

- Population of customers is proportional to the number of stores in the mall:
  \[ X(n + 1) = A \times Y(n) \]
  where A is a multiple of the stores that are open
- Then A * K will be the customer carrying capacity of the mall
The Stores

- The store model based on the discrete logistic growth model is
  \[ Y(n + 1) = Y(n)[1 + \rho(1 - Y(n) / K)] \]
- Where \( \rho \) is the intrinsic growth rate (the rate at which the stores fill the mall)
Minimum Operating Costs

- Electricity
- Insurance
- Snow removal
- Etc.
The Greed Factor (Opportunistic Rent)

- Incorporating the greed factor into the customer model
  \[ X(n + 1) = A \cdot Y(n) - R(X(n), Y(n)) \]
- Where \( R(X, Y) \) represents the customers attrition due to the greed factor
- Let \( R(X(n), Y(n)) = \alpha(n)X(n) + \beta(n)Y(n) \)
- For some positive sequences of \( \{\alpha(n)\}, \{\beta(n)\} \)
Building the Mall Model

**The Customers**

\[ X(n + 1) = A \times Y(n) - \alpha(n)X(n) - \beta(n)Y(n) \]

Where \(- \alpha(n)X(n) - \beta(n)Y(n)\) is customer attrition from last year's price increase.

**The Stores**

\[ Y(n + 2) = Y(n + 1)[1 + \rho (1 - Y(n) / K)] - B(\alpha(n)X(n) + \beta(n)Y(n)) \]

Where the \(B\) is a constant multiplied by the customer attrition in year \(n\).
Behold the Mall Model

Customers:
\[ X(n + 1) = A \times Y(n) - \alpha(n)X(n) - \beta(n)Y(n) \]

Stores:
\[ Y(n + 1) = y(n) \times [1 + \rho (1 - Y(n) / K)] - B(\alpha(n - 1)X(n - 1) - \beta(n - 1)Y(n - 1)) \]
Mall Management & Money

- A large greed factor will produce millions right away = no profits in years to come
- Why?
- Stores have moved or gone out of business, since increase in rent was passed onto customers, whom have gone elsewhere to find lower prices
Mall Viability

- The key to mall viability is a function of the mall managements long term profits

\[ \sum_{n=0}^{24} (\alpha(n)X(n) + \beta(n)Y(n)) \]

- Want \( \alpha \) & \( \beta \) has high as possible without driving stores out and new stores from moving in due to high rent

- Want to find sequences of \( \{\alpha(n)\} \), \( \{\beta(n)\} \) which will maximize this sum
Our Model at Work
Many thanks to

- Manager at Ben Franklin
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- [www.britannica.com](http://www.britannica.com)
- [www.oakwoodmall.com](http://www.oakwoodmall.com)
- [www.mallofamerica.com](http://www.mallofamerica.com)
- And of course, Mr. Deckelman